AP Calculus AB Course Description

The AP Calculus AB course is a one semester class meeting for 90 days, 90 minutes per day. Students who enroll in AB generally go on to take BC in the second semester and take the AP Calculus BC Exam. Conceptual understanding is emphasized, and there is an essential balance in the course between technology and pencil and paper calculations. Each student has his or her own TI-84 plus calculator issued by the school to use for the duration of the course.

Through lecture, independent exploration, practice and collaboration, the students gain an in-depth understanding of the material and make valuable connections between and among topics. At the beginning of each class, the students work together on an introductory problem or activity, discussing ideas and approaches to reach a solution. Students are encouraged to ask questions and engage in class discussions. Homework is collected weekly and graded for completeness and correctness. Students are expected complete out of class assignments and may collaborate on them at the beginning of the next class, allowing students the opportunity to communicate mathematics verbally on a daily basis.

Course Outline

I. Unit 1: Functions and Graphs (Precalculus Review) 6 Days
   a. Linear Functions
   b. Exponential Functions
   c. Logarithmic Functions
   d. Inverse Functions
   e. Parametric Curves
   f. Trigonometric Functions

Teaching Notes:

The focus is on the analysis of graphs and characteristics of functions. Local and global behavior of functions is considered numerically, analytically, and by utilizing the support of graphing technology. Students are expected to make connections between the various representations. Also, the importance of considering all of the representations is emphasized. For example, students are expected to realize that graphical representation is not always sufficient to describe characteristics of functions (e.g. removable discontinuity). From the very beginning of the course, students are encouraged to formulate mathematically sound arguments and reasonable solutions in both verbal and written form.

II. Unit 2: Limits and Continuity (10 Days)

a. Limits
   i. Informal definition of Limits and Notation
   ii. Properties of Limits
   iii. Formal Definition of Limit
   iv. Limits Involving Infinity
   v. Asymptotic Behavior
b. Definition of Continuity
   i. Properties of continuity
   ii. Continuous extension
   iii. Intermediate Value Theorem
c. Tangent Lines
   i. Average Rate of Change
   ii. Instantaneous Rate of Change
   iii. Tangents and Normal lines

Teaching Notes:

Limits are introduced using a graphical, numerical and analytical approach. One-sided limits are also evaluated and explored. Early on, the concept of infinity is incorporated through end behavior and asymptotic behavior of graphs. A strong emphasis is placed on reasons why a limit fails to exist. Graphical exploration is followed by analytical interpretation. Students use the Squeeze (Sandwich) Theorem to solidify their understanding of limits and begin their work with proofs. Verbal or written explanation is required along with their proof.

Continuity is initially approached from a graphical perspective. The idea of proof is revisited with the Intermediate Value Theorem. Through the understanding of local linearity and limits, students construct the concept of the tangent line.

III. Derivatives (14 Days)

a. Definition of Derivative
b. Differentiability
c. Introduction to the relationship between the graphs of f and f'
d. Differentiation Rules
e. Rates of Change
f. Derivatives of Trigonometric Functions
g. Chain Rule
h. Parametric Differentiation
i. Implicit Differentiation
j. Derivatives of Exponential and Logarithmic Functions
k. Derivatives of Inverse Trigonometric Functions

Teaching Notes:

In the beginning of the unit, major emphasis is placed on the relationship between differentiability and continuity. Connections are made between the derivative, tangent line, slope of curve, average rate of change, and instantaneous rate of change. A major emphasis is placed on local linearity, requiring students to discuss the concept in their own words. Graphical exploration is utilized to "zo om-in" on various graphs to determine differentiability. Once conceptual understanding has been accomplished, the focus is shifted to computing derivatives and rules and techniques for differentiation. The differentiation rules are initially explored through the use of graphing technology. Students make conjectures about derivatives of polynomial, trigonometric and exponential functions. After exploration, many of the derivatives of trigonometric, exponential, logarithmic and inverse trigonometric functions are derived by the students using proofs. In particular, proofs of inverse trigonometric functions are written and verbally presented to the class by student teams.

IV. Application of the Derivative (15 days)
  a. Mean and Extreme Value Theorem
  b. Increasing and Decreasing intervals
  c. Concavity
  d. Connecting f with f' and f"
  e. Modeling and Optimization
  f. Linearization
  g. Differentials
  h. Related Rates

Teaching Notes:

This unit is built on two major concepts. In the beginning of the unit, the emphasis is for students to make connections between a function and its first and second derivatives. This is done graphically, algebraically, and numerically. Also, the concept of the derivative as a rate of change is reinforced with reference to position, velocity and acceleration.

The second part of the unit focuses on the application of the derivative to solving problems, such as optimization and related rates problems. Students solve problems analytically, supporting their conclusions with the use of graphing technology. Students gain an appreciation for the usefulness of calculus through real-world scenarios. Also, the concept of approximating functions using tangent lines and differentials is explored graphically, numerically and analytically.

V. Integrals (14 Days)
  a. Riemann Sums

    i. Right
    ii. Left
    iii. Midpoint
b. Definite Integrals
c. Fundamental Theorem of Calculus
d. Trapezoidal Approximation of Definite Integrals
e. Indefinite Integrals
f. Integration Techniques
    i. Integration by Substitution
    ii. Integration by Parts and Tabular Integration

Teaching Notes:

During this unit, there is a strong emphasis of the concept of a definite integral as a numerical value. Approximations involving Riemann Sums and Trapezoids are explored and compared. Right, Left, and Midpoint Riemann Sums and Trapezoidal approximations are done by hand and using programs for the graphing calculator. Students look at indefinite integrals as mathematical objects, mainly general antiderivatives. Integration properties, formulas and techniques are introduced and practiced. Relationships between functions and their antiderivatives are explored using graphing technology, particularly the relationship between position, velocity, and acceleration. One of the major concepts of this unit is the Fundamental Theorem of Calculus, Parts 1 and 2. Students are exposed to a variety of problems involving the FTC and its applications. Students are given excerpts from the College Board's AP Calculus 2005-2006 Professional Development Workshop Materials <u>Special Focus: The Fundamental Theorem of Calculus</u>. Released AB Free Response questions are used to facilitate proper written explanation of mathematical concepts.

VI. Applications of Definite and Indefinite Integrals (22 days)
a. Differential Equations
b. Slope Fields
c. Separable Differential Equations
d. Law of Exponential Change
e. Euler's Method*
f. Partial Fraction Decomposition*
g. Logistic Differential equations*
h. Integrals as Net Change
i. Areas in the Plane
    i. Area under curve
    ii. Area between curves
j. Lengths of Curves*
k. Volumes
    i. Disc method

ii. Washer method
iii. Shell method

Teaching Notes:

In the first part of this unit, the focus is on differential equations, using slope fields as a method to find solution curves and Euler's method as a numerical approximation of the function's value. Students draw slope fields by hand and use a slope field program for the graphing calculators. Discussion is facilitated by use of the overhead projection capabilities of the graphing calculator. Calculations for Euler's method are done by hand using step-by-step iteration and using a calculator program. Students develop written and verbal arguments for whether the solution produced by Euler's method is an over- or under-estimate of the actual solution based on both analytical and graphical analysis. After students have obtained a solid understanding of the concept of a differential equation, separable differential equations are introduced and exponential growth and decay are emphasized.

The second part of the unit focuses on applications of integrals as an accumulation of a rate of change, distance, area and volume. As an introduction to volume, computer-based solids of revolutions are explored at http://college.hmco.com/mathematics/larson/calculus_analytic/7e/students/3dgraphs/ch06.html. Volume is calculated using the disc, washer and shell methods, and using a variety of known cross-sectional areas.

* Indicates BC Topic covered in the first semester

Assessments

Exam: Chapter 1
Quiz: Section 2.1, 2.2
Exam: Chapter 2
Quiz: Sections 3.5 – 3.9
Exam: Chapter 3
Quiz: Section 4.1 – 4.4
Quiz: Section 4.5, 4.6
Exam: Chapter 4
Quiz: Definite Integrals
Exam: Chapter 5
Quiz: Section 6.2, 6.3
Quiz: Section 6.1, 6.4
Exam: Chapter 6
Quiz: Section 7.1, 7.2, 7.4
Exam: Chapter 7
Final Exam

In addition to teacher-generated quizzes and exams, students are also exposed to released Free-response questions from past AB exams and multiple choice practice problems written for AP Exam preparation. These problems are chosen and/or written to facilitate written student explanation of concepts.

Primary textbook

Finney, Demana, Waits, Kennedy

Calculus: Graphical, Numerical, Algebraic (Third Edition) AP Edition
Pearson Prentice Hall. Boston, Massachusetts, 2007.

Supplemental Resources

Textbooks:

Barton, Brunsting, Diehl, Hill, Tyler, Wilson
Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text)
Pearson Education. Boston, Massachusetts, 2007.

Larson, Hostetler, Edwards
Calculus (Sixth edition)
Houghton Mifflin Company. Boston, Massachusetts, 1998.

Websites:

1. AP Central

http://apcentral.collegeboard.com/apc/Controller.jpf

2. Houghton Mifflin: Calculus with Analytic Geometry

http://college.hmco.com/mathematics/larson/calculus_analytic/7e/students/3dgraphs/ch06.html.